

4.3 Writing Equations of Parallel and Perpendicular Lines

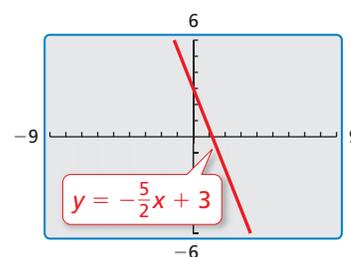
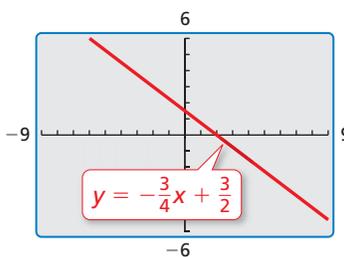
Essential Question How can you recognize lines that are parallel or perpendicular?

EXPLORATION 1 Recognizing Parallel Lines

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear parallel? How can you tell?

a. $3x + 4y = 6$
 $3x + 4y = 12$
 $4x + 3y = 12$

b. $5x + 2y = 6$
 $2x + y = 3$
 $2.5x + y = 5$



USING TOOLS STRATEGICALLY

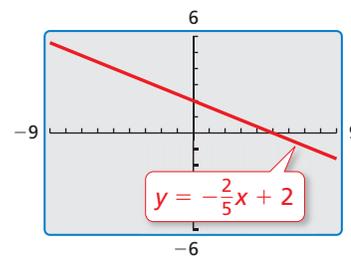
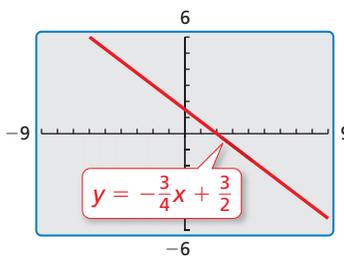
To be proficient in math, you need to use a graphing calculator and other available technological tools, as appropriate, to help you explore relationships and deepen your understanding of concepts.

EXPLORATION 2 Recognizing Perpendicular Lines

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear perpendicular? How can you tell?

a. $3x + 4y = 6$
 $3x - 4y = 12$
 $4x - 3y = 12$

b. $2x + 5y = 10$
 $-2x + y = 3$
 $2.5x - y = 5$



Communicate Your Answer

- How can you recognize lines that are parallel or perpendicular?
- Compare the slopes of the lines in Exploration 1. How can you use slope to determine whether two lines are parallel? Explain your reasoning.
- Compare the slopes of the lines in Exploration 2. How can you use slope to determine whether two lines are perpendicular? Explain your reasoning.

4.3 Lesson

Core Vocabulary

parallel lines, p. 188
perpendicular lines, p. 189

Previous
reciprocal

READING

The phrase “A if and only if B” is a way of writing two conditional statements at once. It means that if A is true, then B is true. It also means that if B is true, then A is true.

What You Will Learn

- ▶ Identify and write equations of parallel lines.
- ▶ Identify and write equations of perpendicular lines.
- ▶ Use parallel and perpendicular lines in real-life problems.

Identifying and Writing Equations of Parallel Lines

Core Concept

Parallel Lines and Slopes

Two lines in the same plane that never intersect are **parallel lines**. Two distinct nonvertical lines are parallel if and only if they have the same slope.

All vertical lines are parallel.

EXAMPLE 1 Identifying Parallel Lines

Determine which of the lines are parallel.

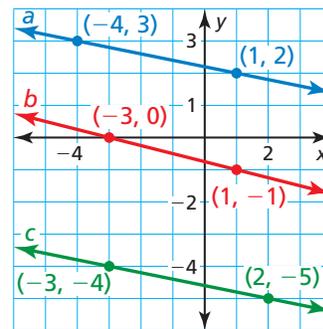
SOLUTION

Find the slope of each line.

$$\text{Line } a: m = \frac{2 - 3}{1 - (-4)} = -\frac{1}{5}$$

$$\text{Line } b: m = \frac{-1 - 0}{1 - (-3)} = -\frac{1}{4}$$

$$\text{Line } c: m = \frac{-5 - (-4)}{2 - (-3)} = -\frac{1}{5}$$



- ▶ Lines a and c have the same slope, so they are parallel.

EXAMPLE 2 Writing an Equation of a Parallel Line

Write an equation of the line that passes through (5, -4) and is parallel to the line $y = 2x + 3$.

SOLUTION

Step 1 Find the slope of the parallel line. The graph of the given equation has a slope of 2. So, the parallel line that passes through (5, -4) also has a slope of 2.

Step 2 Use the slope-intercept form to find the y-intercept of the parallel line.

$$y = mx + b$$

Write the slope-intercept form.

$$-4 = 2(5) + b$$

Substitute 2 for m , 5 for x , and -4 for y .

$$-14 = b$$

Solve for b .

- ▶ Using $m = 2$ and $b = -14$, an equation of the parallel line is $y = 2x - 14$.

ANOTHER WAY

You can also use the slope $m = 2$ and the point-slope form to write an equation of the line that passes through (5, -4).

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = 2(x - 5)$$

$$y = 2x - 14$$

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1. Line a passes through $(-5, 3)$ and $(-6, -1)$. Line b passes through $(3, -2)$ and $(2, -7)$. Are the lines parallel? Explain.
2. Write an equation of the line that passes through $(-4, 2)$ and is parallel to the line $y = \frac{1}{4}x + 1$.

Identifying and Writing Equations of Perpendicular Lines

REMEMBER

The product of a nonzero number m and its negative reciprocal is -1 :

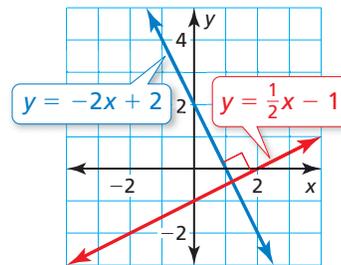
$$m\left(-\frac{1}{m}\right) = -1.$$

Core Concept

Perpendicular Lines and Slopes

Two lines in the same plane that intersect to form right angles are **perpendicular lines**. Nonvertical lines are perpendicular if and only if their slopes are negative reciprocals.

Vertical lines are perpendicular to horizontal lines.



EXAMPLE 3 Identifying Parallel and Perpendicular Lines

Determine which of the lines, if any, are parallel or perpendicular.

Line a : $y = 4x + 2$

Line b : $x + 4y = 3$

Line c : $-8y - 2x = 16$

SOLUTION

Write the equations in slope-intercept form. Then compare the slopes.

Line a : $y = 4x + 2$

Line b : $y = -\frac{1}{4}x + \frac{3}{4}$

Line c : $y = -\frac{1}{4}x - 2$

- Lines b and c have slopes of $-\frac{1}{4}$, so they are parallel. Line a has a slope of 4 , the negative reciprocal of $-\frac{1}{4}$, so it is perpendicular to lines b and c .

EXAMPLE 4 Writing an Equation of a Perpendicular Line

Write an equation of the line that passes through $(-3, 1)$ and is perpendicular to the line $y = \frac{1}{2}x + 3$.

SOLUTION

Step 1 Find the slope of the perpendicular line. The graph of the given equation has a slope of $\frac{1}{2}$. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line that passes through $(-3, 1)$ is -2 .

Step 2 Use the slope $m = -2$ and the point-slope form to write an equation of the perpendicular line that passes through $(-3, 1)$.

$$y - y_1 = m(x - x_1)$$

Write the point-slope form.

$$y - 1 = -2[x - (-3)]$$

Substitute -2 for m , -3 for x_1 , and 1 for y_1 .

$$y - 1 = -2x - 6$$

Simplify.

$$y = -2x - 5$$

Write in slope-intercept form.

- An equation of the perpendicular line is $y = -2x - 5$.

ANOTHER WAY

You can also use the slope $m = -2$ and the slope-intercept form to write an equation of the line that passes through $(-3, 1)$.

$$y = mx + b$$

$$1 = -2(-3) + b$$

$$-5 = b$$

So, $y = -2x - 5$.

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3. Determine which of the lines, if any, are parallel or perpendicular. Explain.

Line a : $2x + 6y = -3$ Line b : $y = 3x - 8$ Line c : $-6y + 18x = 9$

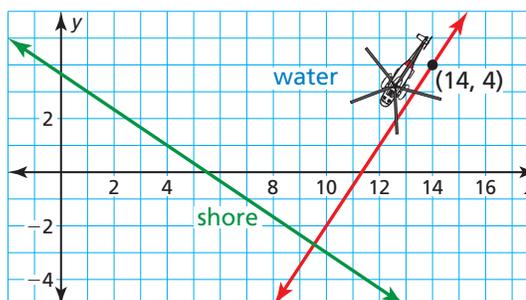
4. Write an equation of the line that passes through $(-3, 5)$ and is perpendicular to the line $y = -3x - 1$.

Writing Equations for Real-Life Problems

EXAMPLE 5 Writing an Equation of a Perpendicular Line



The position of a helicopter search and rescue crew is shown in the graph. The shortest flight path to the shoreline is one that is perpendicular to the shoreline. Write an equation that represents this path.



SOLUTION

- Understand the Problem** You can see the line that represents the shoreline. You know the coordinates of the helicopter. You are asked to write an equation that represents the shortest flight path to the shoreline.
- Make a Plan** Find the slope of the line that represents the shoreline. Use the negative reciprocal of this slope, the coordinates of the helicopter, and the point-slope form to write an equation.
- Solve the Problem**

Step 1 Find the slope of the line that represents the shoreline. The line passes through points (1, 3) and (4, 1). So, the slope is

$$m = \frac{1 - 3}{4 - 1} = -\frac{2}{3}$$

Because the shoreline and shortest flight path are perpendicular, the slopes of their respective graphs are negative reciprocals. So, the slope of the graph of the shortest flight path is $\frac{3}{2}$.

Step 2 Use the slope $m = \frac{3}{2}$ and the point-slope form to write an equation of the shortest flight path that passes through (14, 4).

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Write the point-slope form.} \\ y - 4 &= \frac{3}{2}(x - 14) && \text{Substitute } \frac{3}{2} \text{ for } m, 14 \text{ for } x_1, \text{ and } 4 \text{ for } y_1. \\ y - 4 &= \frac{3}{2}x - 21 && \text{Distributive Property} \\ y &= \frac{3}{2}x - 17 && \text{Write in slope-intercept form.} \end{aligned}$$

► An equation that represents the shortest flight path is $y = \frac{3}{2}x - 17$.

- Look Back** To check that your equation is correct, verify that (14, 4) is a solution of the equation.

$$4 = \frac{3}{2}(14) - 17 \quad \checkmark$$

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- In Example 5, a boat is traveling parallel to the shoreline and passes through (9, 3). Write an equation that represents the path of the boat.

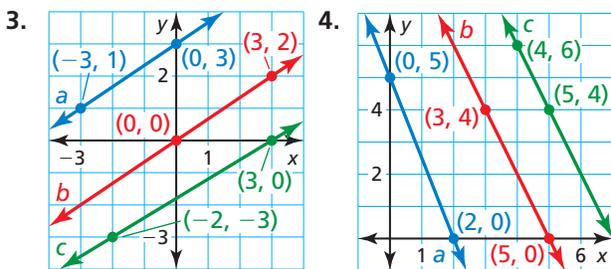
4.3 Exercises

Vocabulary and Core Concept Check

- COMPLETE THE SENTENCE** Two distinct nonvertical lines that have the same slope are _____.
- VOCABULARY** Two lines are perpendicular. The slope of one line is $-\frac{5}{7}$. What is the slope of the other line? Justify your answer.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine which of the lines, if any, are parallel. Explain. (See Example 1.)

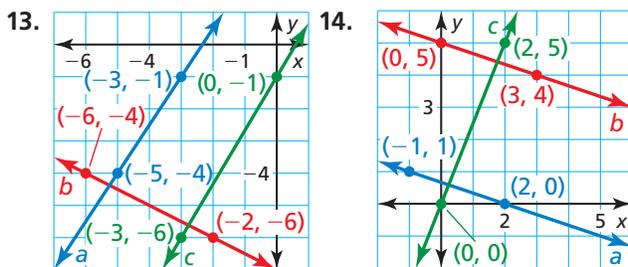


- Line a passes through $(-1, -2)$ and $(1, 0)$.
Line b passes through $(4, 2)$ and $(2, -2)$.
Line c passes through $(0, 2)$ and $(-1, 1)$.
- Line a passes through $(-1, 3)$ and $(1, 9)$.
Line b passes through $(-2, 12)$ and $(-1, 14)$.
Line c passes through $(3, 8)$ and $(6, 10)$.
- Line a : $4y + x = 8$ 8. Line a : $3y - x = 6$
Line b : $2y + x = 4$ Line b : $3y = x + 18$
Line c : $2y = -3x + 6$ Line c : $3y - 2x = 9$

In Exercises 9–12, write an equation of the line that passes through the given point and is parallel to the given line. (See Example 2.)

- $(-1, 3)$; $y = 2x + 2$ 10. $(1, 2)$; $y = -5x + 4$
- $(18, 2)$; $3y - x = -12$ 12. $(2, -5)$; $2y = 3x + 10$

In Exercises 13–18, determine which of the lines, if any, are parallel or perpendicular. Explain. (See Example 3.)

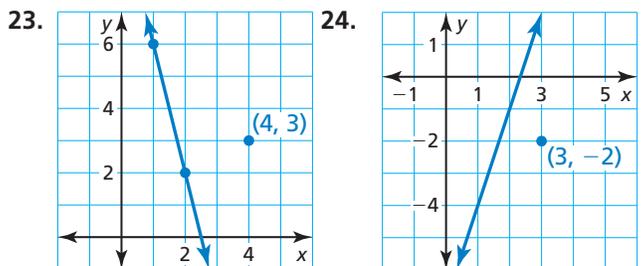


- Line a passes through $(-2, 1)$ and $(0, 3)$.
Line b passes through $(4, 1)$ and $(6, 4)$.
Line c passes through $(1, 3)$ and $(4, 1)$.
- Line a passes through $(2, 10)$ and $(4, 13)$.
Line b passes through $(4, 9)$ and $(6, 12)$.
Line c passes through $(2, 10)$ and $(4, 9)$.
- Line a : $4x - 3y = 2$ 18. Line a : $y = 6x - 2$
Line b : $y = \frac{4}{3}x + 2$ Line b : $6y = -x$
Line c : $4y + 3x = 4$ Line c : $y + 6x = 1$

In Exercises 19–22, write an equation of the line that passes through the given point and is perpendicular to the given line. (See Example 4.)

- $(7, 10)$; $y = \frac{1}{2}x - 9$ 20. $(-4, -1)$; $y = \frac{4}{3}x + 6$
- $(-3, 3)$; $2y = 8x - 6$ 22. $(8, 1)$; $2y + 4x = 12$

In Exercises 23 and 24, write an equation of the line that passes through the given point and is (a) parallel and (b) perpendicular to the given line.



- ERROR ANALYSIS** Describe and correct the error in writing an equation of the line that passes through $(1, 3)$ and is parallel to the line $y = \frac{1}{4}x + 2$.

X

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -4(x - 1)$$

$$y - 3 = -4x + 4$$

$$y = -4x + 7$$

26. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line that passes through $(4, -5)$ and is perpendicular to the line $y = \frac{1}{3}x + 5$.

X

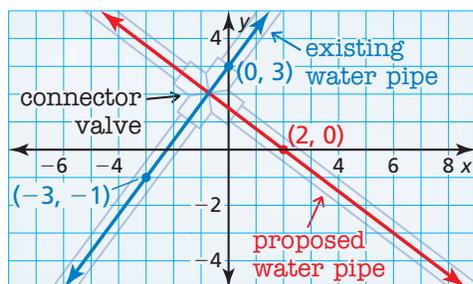
$$y - y_1 = m(x - x_1)$$

$$y - (-5) = 3(x - 4)$$

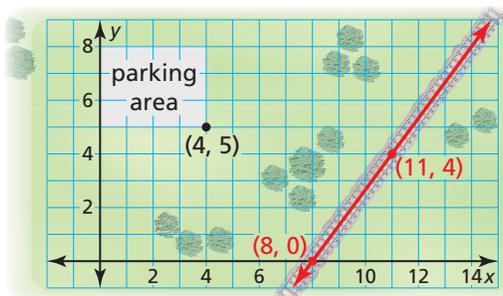
$$y + 5 = 3x - 12$$

$$y = 3x - 17$$

27. **MODELING WITH MATHEMATICS** A city water department is proposing the construction of a new water pipe, as shown. The new pipe will be perpendicular to the old pipe. Write an equation that represents the new pipe. (See Example 5.)



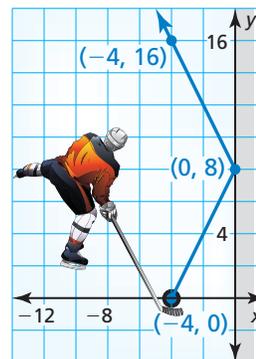
28. **MODELING WITH MATHEMATICS** A parks and recreation department is constructing a new bike path. The path will be parallel to the railroad tracks shown and pass through the parking area at the point $(4, 5)$. Write an equation that represents the path.



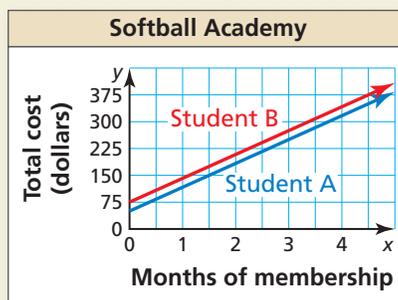
29. **MATHEMATICAL CONNECTIONS** The vertices of a quadrilateral are $A(2, 2)$, $B(6, 4)$, $C(8, 10)$, and $D(4, 8)$.
- Is quadrilateral $ABCD$ a parallelogram? Explain.
 - Is quadrilateral $ABCD$ a rectangle? Explain.
30. **USING STRUCTURE** For what value of a are the graphs of $6y = -2x + 4$ and $2y = ax - 5$ parallel? perpendicular?

31. **MAKING AN ARGUMENT**

A hockey puck leaves the blade of a hockey stick, bounces off a wall, and travels in a new direction, as shown. Your friend claims the path of the puck forms a right angle. Is your friend correct? Explain.



32. **HOW DO YOU SEE IT?** A softball academy charges students an initial registration fee plus a monthly fee. The graph shows the total amounts paid by two students over a 4-month period. The lines are parallel.



- Did one of the students pay a greater registration fee? Explain.
- Did one of the students pay a greater monthly fee? Explain.

REASONING In Exercises 33–35, determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

- Two lines with positive slopes are perpendicular.
- A vertical line is parallel to the y -axis.
- Two lines with the same y -intercept are perpendicular.

36. **THOUGHT PROVOKING** You are designing a new logo for your math club. Your teacher asks you to include at least one pair of parallel lines and at least one pair of perpendicular lines. Sketch your logo in a coordinate plane. Write the equations of the parallel and perpendicular lines.

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Determine whether the relation is a function. Explain. (Section 3.1)

37. $(3, 6), (4, 8), (5, 10), (6, 9), (7, 14)$ 38. $(-1, 6), (1, 4), (-1, 2), (1, 6), (-1, 5)$